Econometrics

Practical Session 19

Nonstationarity: Structural Breaks

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Theoretical Wrap-up

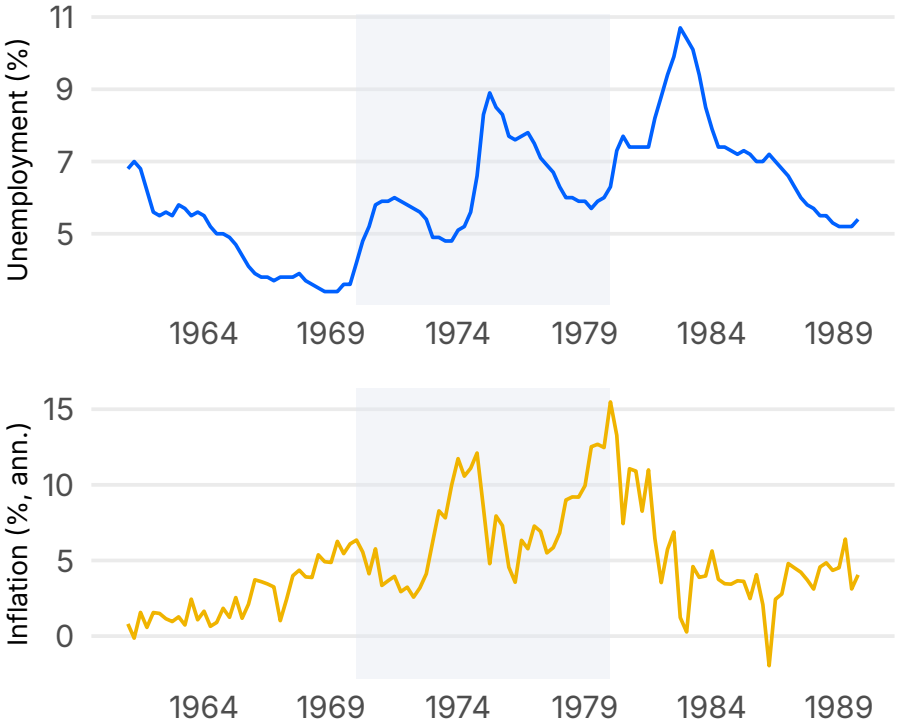
A Second Type of Nonstationarity: Structural Breaks

- So far, nonstationarity meant **unit roots**
- An equally important type: **the regression coefficients change over time**
- **Why does this matter for forecasting?**
  - Our models are trained on **historical data → estimates no longer apply**
  - This is an **external validity** problem: the model doesn't generalize beyond the training period

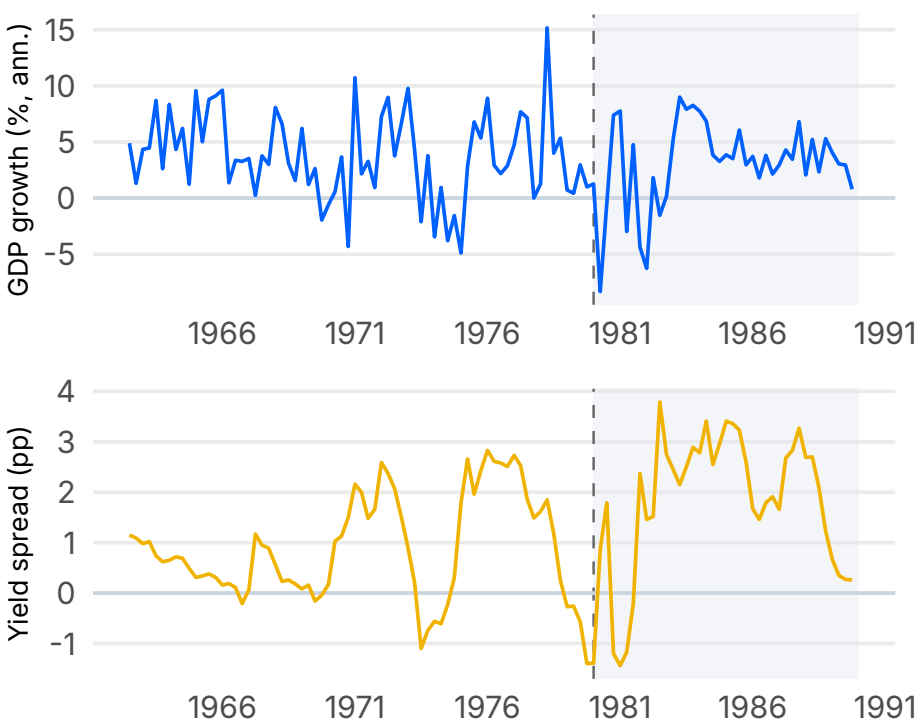
A Second Type of Nonstationarity: Structural Breaks

- **Examples:**
  - The Phillips curve (inflation-unemployment) appeared stable in the 1960s but broke down in the 1970s
  - The GDP-yield spread relationship was disrupted around 1980 (Volcker disinflation)
  - Financial models estimated before 2008 missed the crisis
- **Informal diagnostic:** do pseudo out-of-sample forecasts track actual values at the end of the sample?

A Second Type of Nonstationarity: Structural Breaks



Source: FRED, series UNRATE, CPIAUCSL. Gray band: 1970s.



Source: FRED (via us_quarterly.csv). Dashed line: 1980Q1 (Volcker).

Case I: Known Break Date — The Chow Test

- Suppose a break at date τ is **known in advance** (e.g., a policy change, a law, a major event)
- **Strategy:** estimate a **fully interacted regression** → allow all coefficients to differ before and after τ:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \delta_1 X_{t-1} + \gamma_0 D_{t(\tau)} + \gamma_1 [D_{t(\tau)} \times Y_{t-1}] + \gamma_2 [D_{t(\tau)} \times X_{t-1}] + u_t$$

  where $D_{t(\tau)} = 1$ if $t \geq \tau$, else 0
- **Chow test:** F-test of $H_0 : \gamma_0 = \gamma_1 = \gamma_2 = 0$ → if we reject: the regression function changed at τ → break confirmed
- **Problem:** in practice we often do **not** know τ in advance

Case II: Unknown Break Date — The QLR Test

- **Data snooping:** plot the data, notice a kink in 1980, and run a Chow test
  - A Chow test at a **pre-specified τ** has a 5% false-rejection rate under the null → $F(\tau)$ follows a standard F distribution
  - But you **implicitly picked the one where F is highest** → you are **testing the maximum**, not a single statistic
  - The maximum of many **F-statistics is larger than any single one**, even with **no break** → so the **standard critical values rejects far more** than 5% of the time
- **Solution:** run a Chow $F(\tau)$ at **every candidate date** in the trimmed range, then **report the maximum** and use **higher critical values**

Case II: Unknown Break Date — The QLR Test

- The **Quandt Likelihood Ratio (QLR)** (*a.k.a.* sup-Wald) statistic:

$$\text{QLR} = \max_{\tau \in [\tau_0, \tau_1]} F(\tau)$$

  where $F(\tau)$ is the Chow F-statistic for a break at date τ

- **The trimmings** guarantee a minimum **usable sub-samples before or after the candidate date** if that date is close to beginning or the end of the sample

QLR Critical Values

- The QLR has a non-standard distribution: critical values depend on the **number of restrictions q and the trimming fraction**
- Critical values from S&W Table 15.5 (15% trimming):

Restrictions q	10%	5%	1%
1	7.12	8.68	12.16
2	5.00	5.86	7.78
3	4.09	4.71	6.02
4	3.59	4.09	5.12
5	3.26	3.66	4.53

QLR Critical Values

Restrictions q	10%	5%	1%
6	3.02	3.37	4.12
7	2.84	3.15	3.82
8	2.69	2.98	3.57
9	2.58	2.84	3.38
10	2.48	2.71	3.23

Informal Diagnostic: POOS Forecasting

- The **QLR test has low power** toward the **end of the sample**, because of trimming
- But end-of-sample breaks are often the most practically important!
- **Pseudo out-of-sample (POOS) forecasting** as a complement:
  1. Estimate the model on a growing training window (recursive estimation)
  2. At each t, forecast $t + 1$ using only data up to t
  3. Plot actual vs. forecast: does the model track recently?

Informal Diagnostic: POOS Forecasting

- **What to look for:**
  - If forecasts systematically under- or over-shoot in recent observations → likely end-of-sample break
  - If errors in recent periods are much larger than earlier → model may have changed
- This is an **informal diagnostic**, not a formal test → complement it with QLR

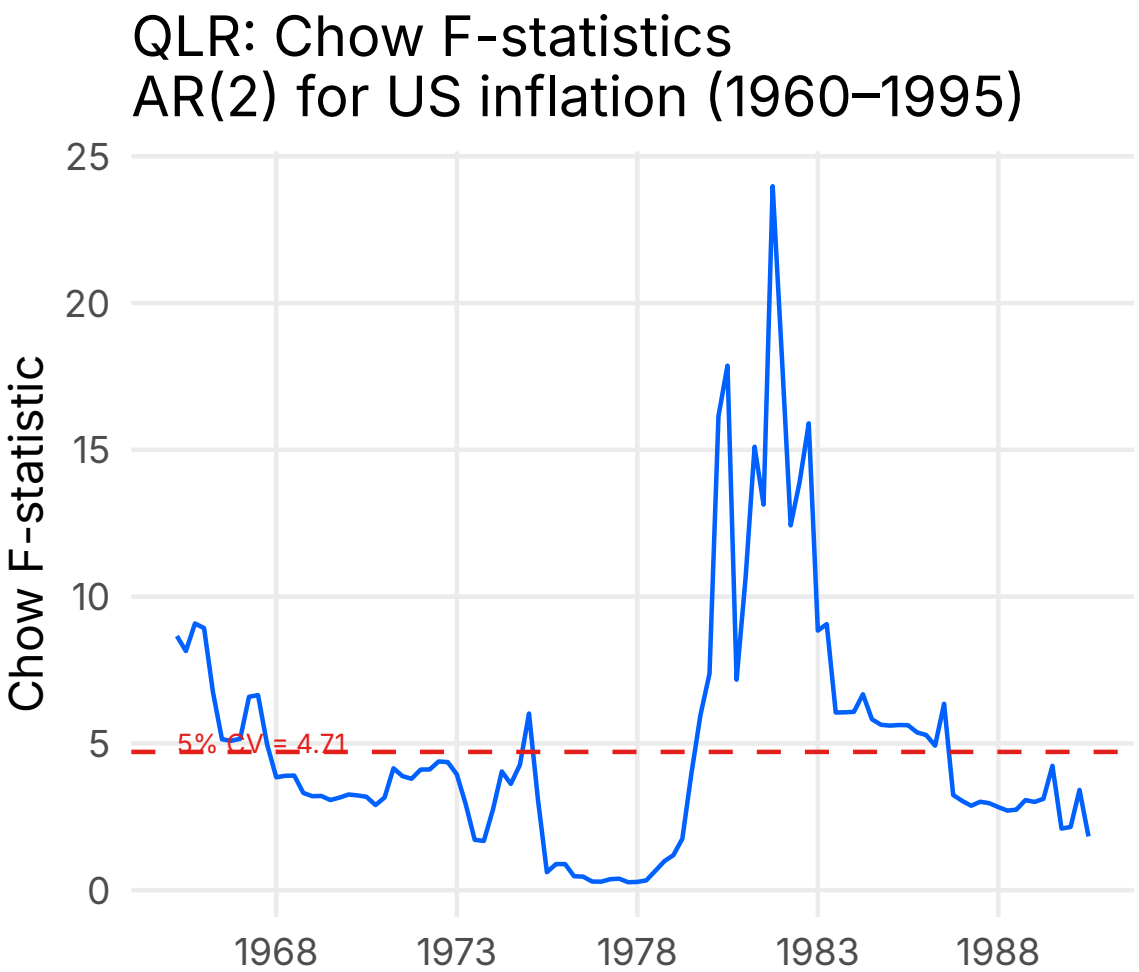
Exercises

Exercise 1 | A Clean Structural Break: US Inflation (1960–1995)

Using quarterly CPI data (FRED: CPIAUCSL), we fit an AR(2) to annualised US inflation over 1960Q1–1995Q4 and apply the QLR test.

The Chow $F(\tau)$ statistics over the trimmed sample are plotted below. The dashed line marks the 5% critical value for $q = 3$ (intercept + 2 lags).

Exercise 1 | A Clean Structural Break: US Inflation (1960–1995)



Is there evidence of a structural break? Does the plot show a **single clean break** or **pervasive instability**? At what approximate date does the maximum occur, and what economic event explains it?

Exercise 2 | Testing for a Structural Break in the ADL(2,2)

Using `us_quarterly.csv` and the ADL(2,2) model for GDP growth and the term spread (estimated in Session 16), we test whether the GDP–spread relationship has been stable over time.

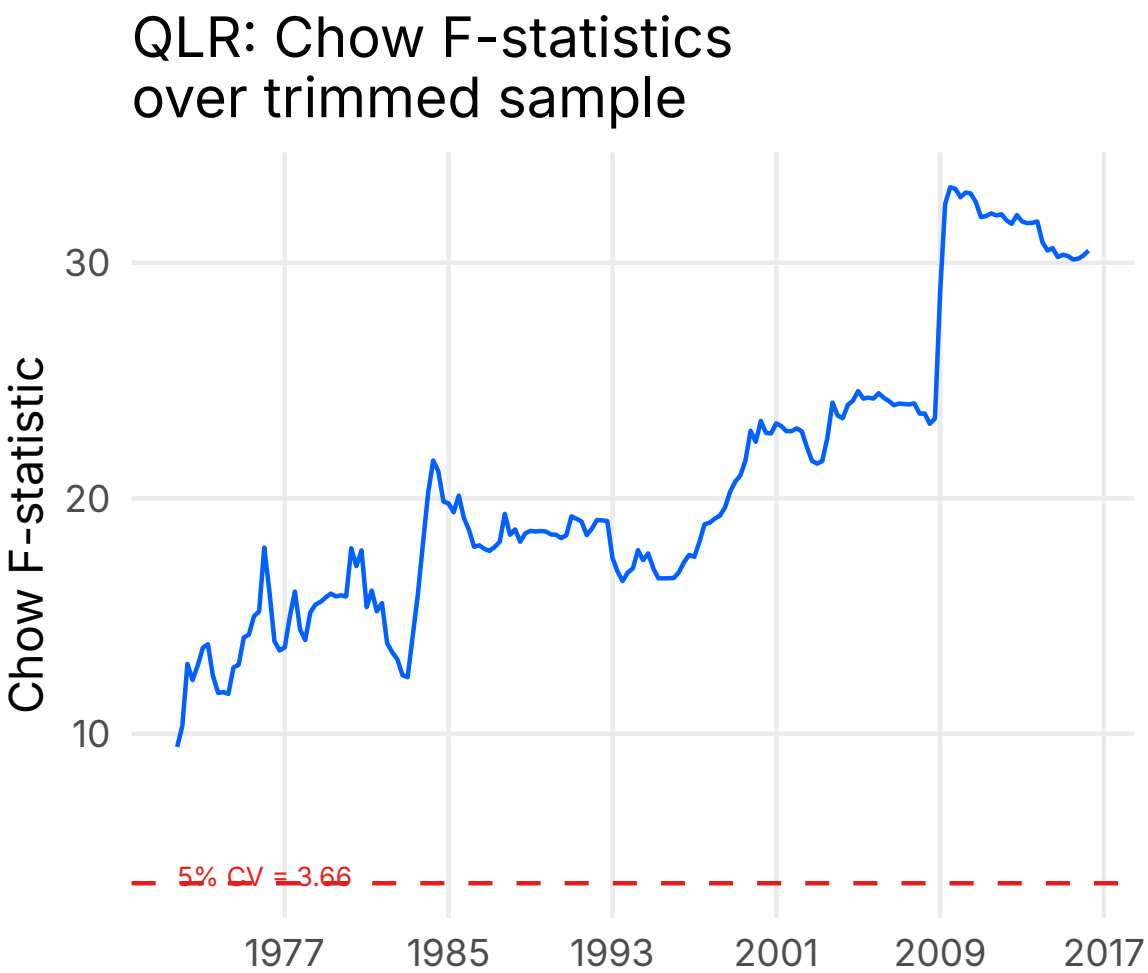
a) The ADL(2,2) is estimated on the full sample (1962Q3–2026Q1). The QLR test with 15% trimming gives:

```
supF test

data:  GR ~ GR_l1 + GR_l2 + Spread_l1 + Spread_l2
sup.F = 33.08,   p-value < 0.001
```

The 5% critical value for $q = 5$ from S&W Table 15.5 is **3.66**. Is there evidence of a structural break? What does a very small p-value tell you here?

Exercise 2 | Testing for a Structural Break in the ADL(2,2)



b) The sequence of Chow $F(\tau)$ statistics over the trimmed sample is plotted on the left. The dashed line marks the 5% critical value. At what approximate date does the maximum occur? What economic event might explain a break near that date?

Exercise 2 | Testing for a Structural Break in the ADL(2,2)

c) The ADL(2,2) is re-estimated on the pre-1980 (1962Q1–1979Q4) and post-1980 (1980Q1–2025Q4) sub-samples. The spread coefficients are:

Sub-sample	$\hat{\beta}_{Spread,1}$	$\hat{\beta}_{Spread,2}$
Pre-1980 (n ≈ 70)	2.21 (1.06)	−0.94 (1.16)
Post-1980 (n ≈ 180)	−0.96 (0.56)	1.45 (0.56)

How do the spread coefficients differ across sub-samples? Which sub-sample would you use for forecasting 2026Q1? What principle guides this choice?

Exercise 2 | Testing for a Structural Break in the ADL(2,2)

d) A Chow test for a **known** break at the COVID recession (2020Q2), using a fully interacted regression, gives:

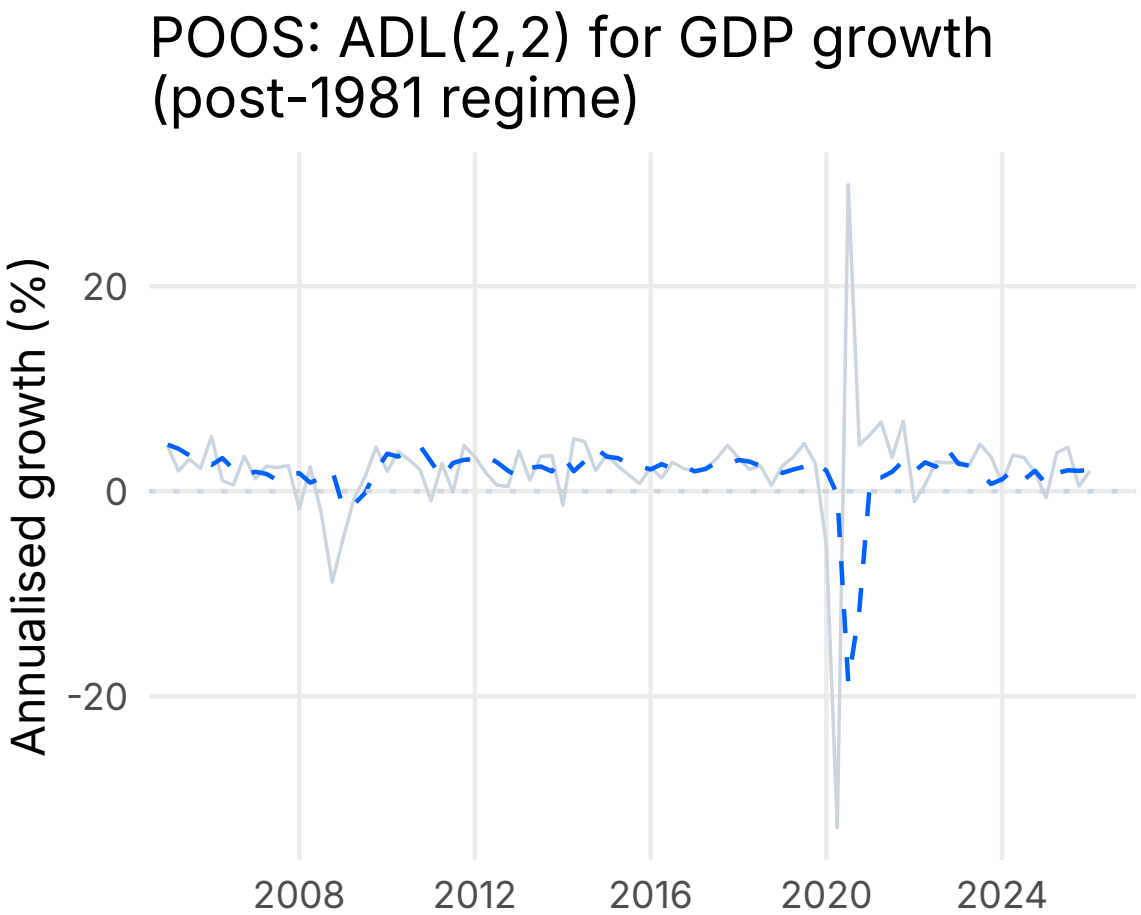
```
Linear hypothesis test

Hypothesis:
D = 0,  GR_l1_D = 0,  GR_l2_D = 0,  Spread_l1_D = 0,  Spread_l2_D = 0

F = 6.06,   Df = 5,   p-value < 0.001
```

Is there evidence that the ADL coefficients changed after COVID? What does this imply for the post-1980 model you chose in c)?

Exercise 3 | POOS Analysis



Gray: actual. Blue dashed: pseudo out-of-sample forecast.

a) Using a rolling training window from 1981Q1, pseudo out-of-sample one-step-ahead forecasts are computed for 2005Q1–2025Q4 and plotted on the left. Describe what you see. Does the model track actual GDP growth well across the full hold-out period?

Exercise 3 | POOS Analysis

b) What do the POOS forecast errors at the 2008–09 financial crisis and the COVID recession tell us about model stability?

Exercise 3 | POOS Analysis

c) The POOS RMSFE over the full hold-out period (2005Q1–2024Q4) is **6.98 pp**. When the crisis quarters (2008–09 and 2020) are excluded, it falls to **1.89 pp**. What does this tell you about where the forecast errors are concentrated? Does the model perform well in normal times?

Discussion

1. The QLR test is designed to detect a single structural break. What would you do if you suspected **multiple** breaks over a long sample (e.g., 1960–2025)?

Discussion

2. What steps should you follow to implement a good time series modelling strategy?

Discussion

3. The COVID quarter (2020Q2) showed GDP growth of approximately –31%. One student says: *"This is a structural break — we should split the sample at 2020Q2."* Another says: *"This is a one-off outlier — we should add a dummy variable for that quarter and keep the full sample."* Who is right?