Econometrics

Practical Session 18

Nonstationarity: Deterministic and Stochastic Trends

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Theoretical Wrap-up

What Is a Trend?

- A **trend** is a persistent, long-term movement in a time series
- Two fundamentally different types:

Deterministic trend

$$y_t = \alpha + \beta t + \varepsilon_t$$

- Trend is a fixed function of time
- ε_t is stationary
- Shocks are **temporary**: y_t reverts to $\alpha + \beta t$ after a shock

Stochastic trend (random walk)

$$y_t = y_{t-1} + \varepsilon_t$$

- Trend is random
- Shocks are **permanent**: each ε_t shifts the level forever → no fixed mean to revert to

- Both introduce **non-stationarity** → each has a particular remedy

Deterministic vs Stochastic Trends: Moments

Deterministic trend

$$y_t = \alpha + \beta t + \varepsilon_t$$

- $\mathbb{E}[y_t] = \alpha + \beta t$
- $\mathrm{var}(y_t) = \sigma_u^2$ (stable)

Random walk

$$y_t = y_{t-1} + \varepsilon_t$$

- $\mathbb{E}[y_t] = y_0$
- $\mathrm{var}(y_t) = t\sigma^2 \to +\infty$

RW with drift

$$y_t = \mu + y_{t-1} + \varepsilon_t$$

- $\mathbb{E}[y_t] = y_0 + \mu t$
- $\mathrm{var}(y_t) = t\sigma^2 \to +\infty$

- **Unit root** ($\beta_1 = 1$ in AR(1)) $\Leftrightarrow$ stochastic trend; shocks accumulate permanently
- **No unit root** ($|\beta_1| < 1$) $\Leftrightarrow$ deterministic trend or stationary; history fades

Problems Caused by Stochastic Trends

1. **Biased AR coefficients:** OLS estimates of β_1 in an AR(1) are biased toward 1 → forecasts are unreliable
2. **Invalid inference:** t-statistics do not follow the standard normal distribution even in large samples
3. **Spurious regression:** regress two **independent** random walks on each other:
    - OLS gives highly significant coefficients and R^2 close to 1
    - The result is pure noise → "trending together" in a plot is **not** evidence of a relationship

- **Rule:** check for stationarity before any regression or AR model

The Dickey-Fuller and Augmented Dickey-Fuller Tests

- Consider that:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t \Leftrightarrow \Delta y_t = \beta_0 + \underbrace{(\beta_1 - 1)}_{\delta} y_{t-1} + \varepsilon_t$$

- **Null hypothesis:** $H_0 : \delta = 0$ (i.e. $\beta_1 = 1$, unit root)
  Alternative: $H_1 : \delta < 0$ (i.e. $\beta_1 < 1$, mean-reverting) → one-sided
- **Critical values follow a non-standard distribution** → Dickey-Fuller table
- **Augmented DF (ADF):** add **lagged differences** to absorb serial correlation in ε_t, **choosing p by BIC or by the residuals ACF** (so that they become white noise):

$$\Delta y_t = \beta_0 + \delta y_{t-1} + \gamma_1 \Delta y_{t-1} + \ldots + \gamma_{p-1} \Delta y_{t-p+1} + \varepsilon_t$$

ADF: Intercept Only vs. Intercept + Trend

Two test regressions, with **different critical values:**

(a) Intercept only

$$\Delta y_t = \beta_0 + \delta y_{t-1} + \ldots + \varepsilon_t$$

- Alternative: y_t stationary around a **constant**
- Use when the series has **no visible long-run trend**

(b) Intercept + time trend

$$\Delta y_t = \beta_0 + \mu t + \delta y_{t-1} + \ldots + \varepsilon_t$$

- Alternative: y_t stationary around a **linear trend**
- Use when the series **trends upward or downward** over time

- **Rule of thumb:** look at the data first → if y_t trends, include t
  → Misspecification matters

Dickey-Fuller Critical Values

Significance	(a) Intercept only	(b) Intercept + trend
1%	−3.43	−3.96
5%	−2.86	−3.41
10%	−2.57	−3.12

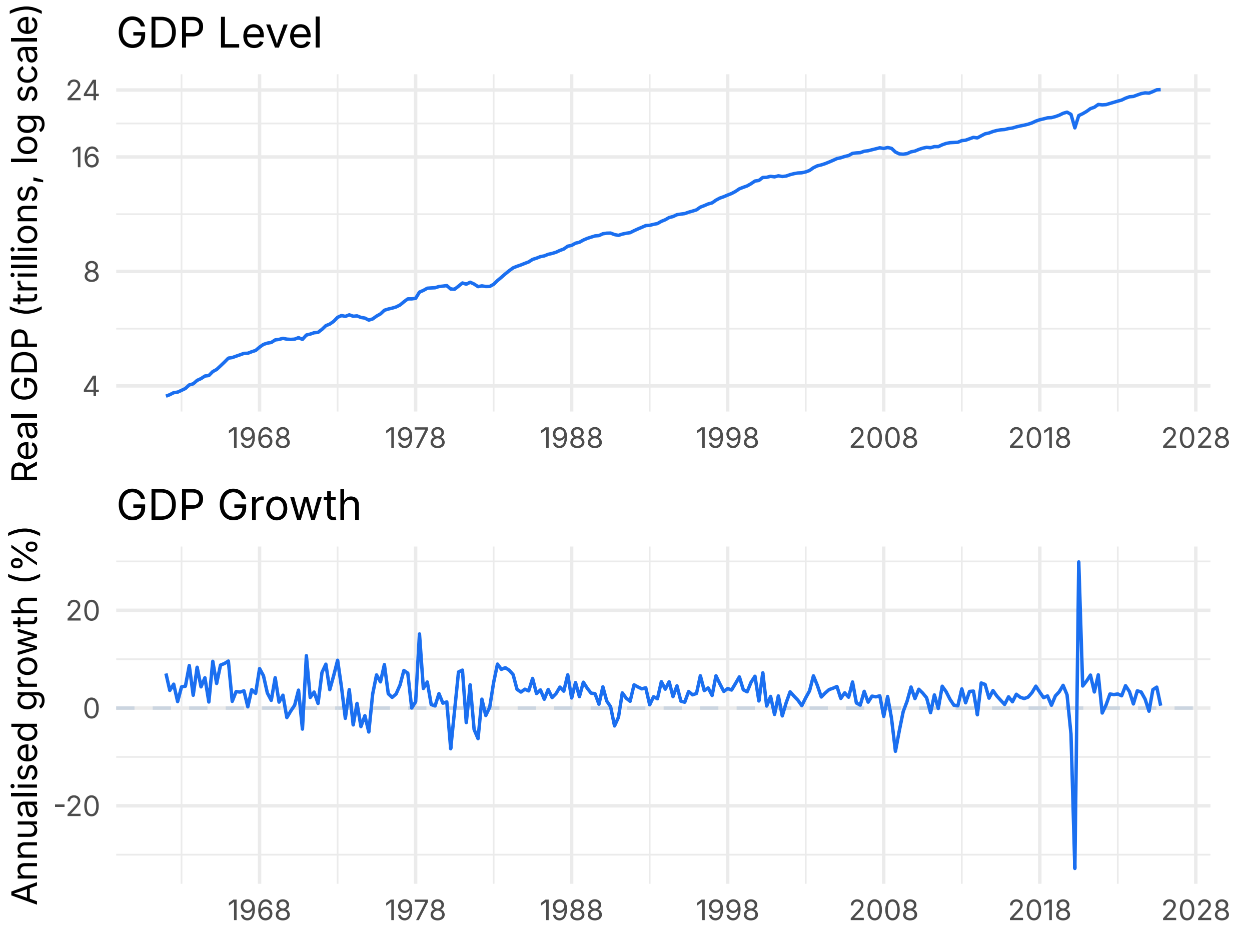
Asymptotic critical values. Reject H_0 if $\tau <$ critical value.
Source: MacKinnon (1991), Stock & Watson Table 15.4.

Achieving Stationarity: First Differencing

- If y_t has a unit root, its **first difference** $\Delta y_t = y_t - y_{t-1}$ is typically stationary
- **Integrated of order 1** ($I(1)$): y_t nonstationary, Δy_t stationary
  - GDP **level** is $I(1)$; GDP **growth** $= 400 \times \Delta \ln \text{GDP}_t$ is $I(0)$
  - CPI **level** is $I(1)$; inflation $= 400 \times \Delta \ln \text{CPI}_t$ is $I(0)$
- **Practical advice:** if y_t has a stochastic trend, model Δy_t → not y_t in levels
- If y_t has a **deterministic** trend, subtract the trend (detrend) rather than differencing
- **Do not difference unnecessarily:** over-differencing induces spurious MA structure and destroys long-run information

Data Overview

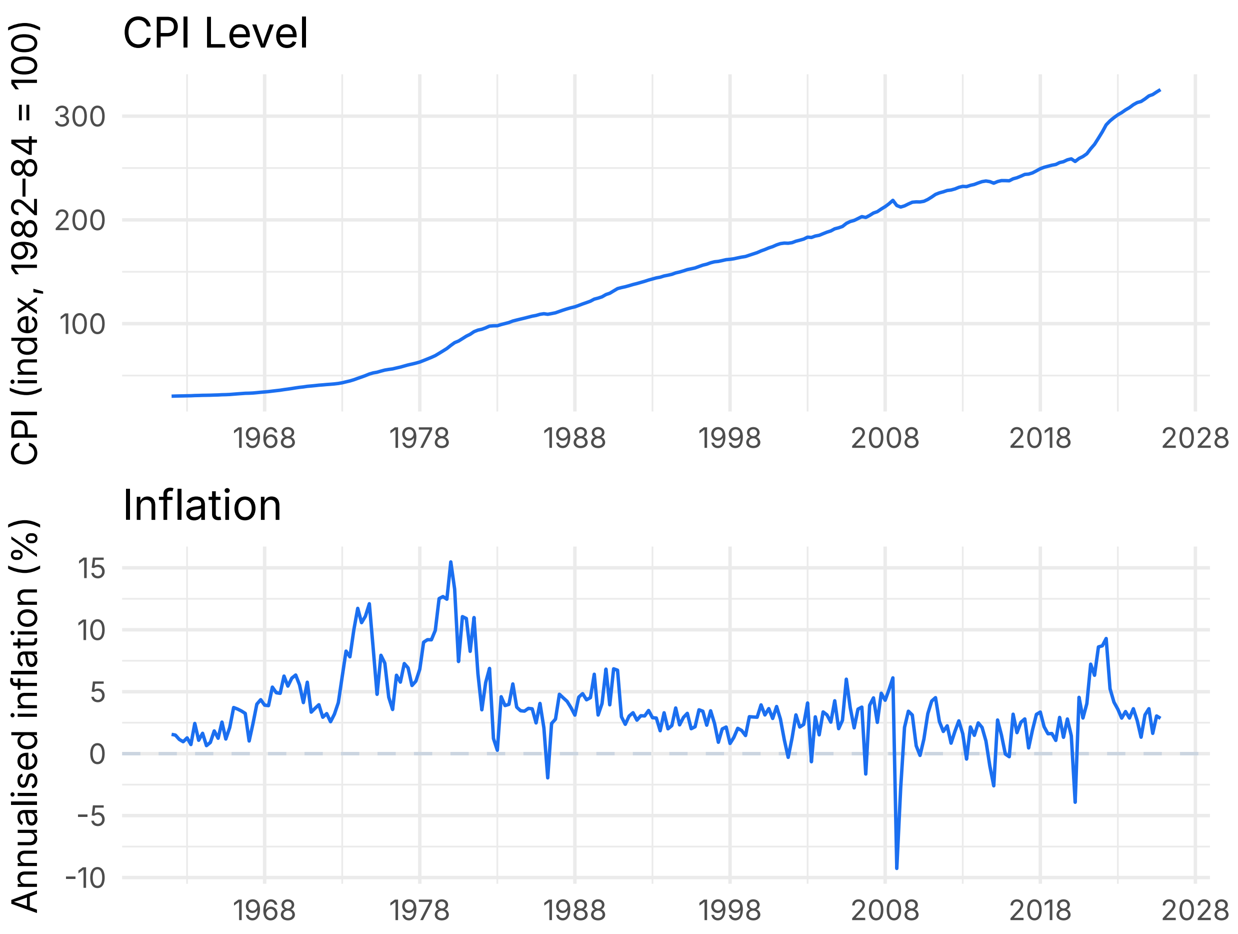
Real GDP and GDP Growth



Source: FRED, series GDPC1.

- **Real GDP level** (top): billions of chained 2017 USD, log scale
- **GDP growth** (bottom): annualised quarterly growth rate ($400 \times \Delta \ln \mathrm{GDP}_t$)
- Quarterly data, 1962–2025

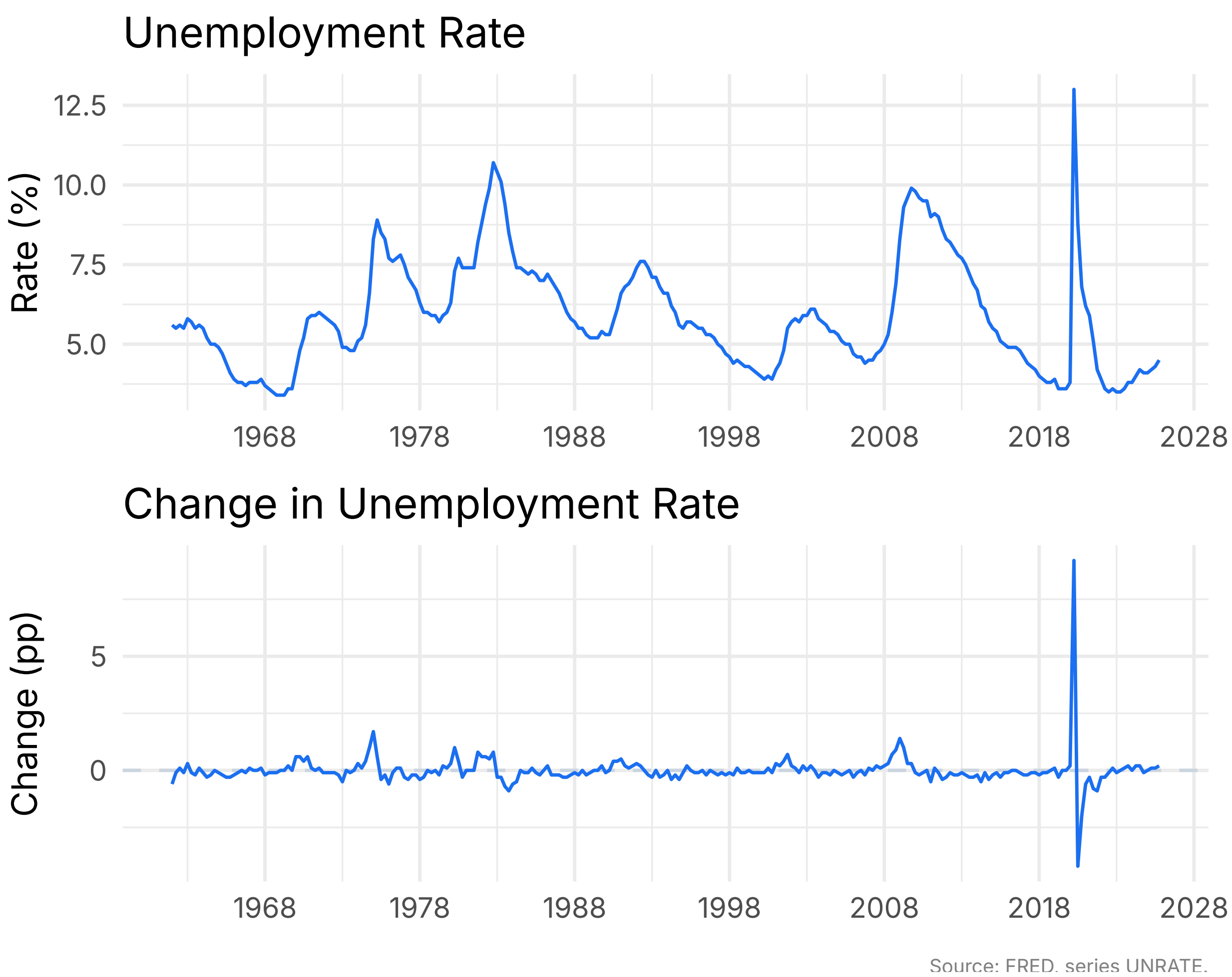
CPI and Inflation



Source: FRED, series CPIAUCSL.

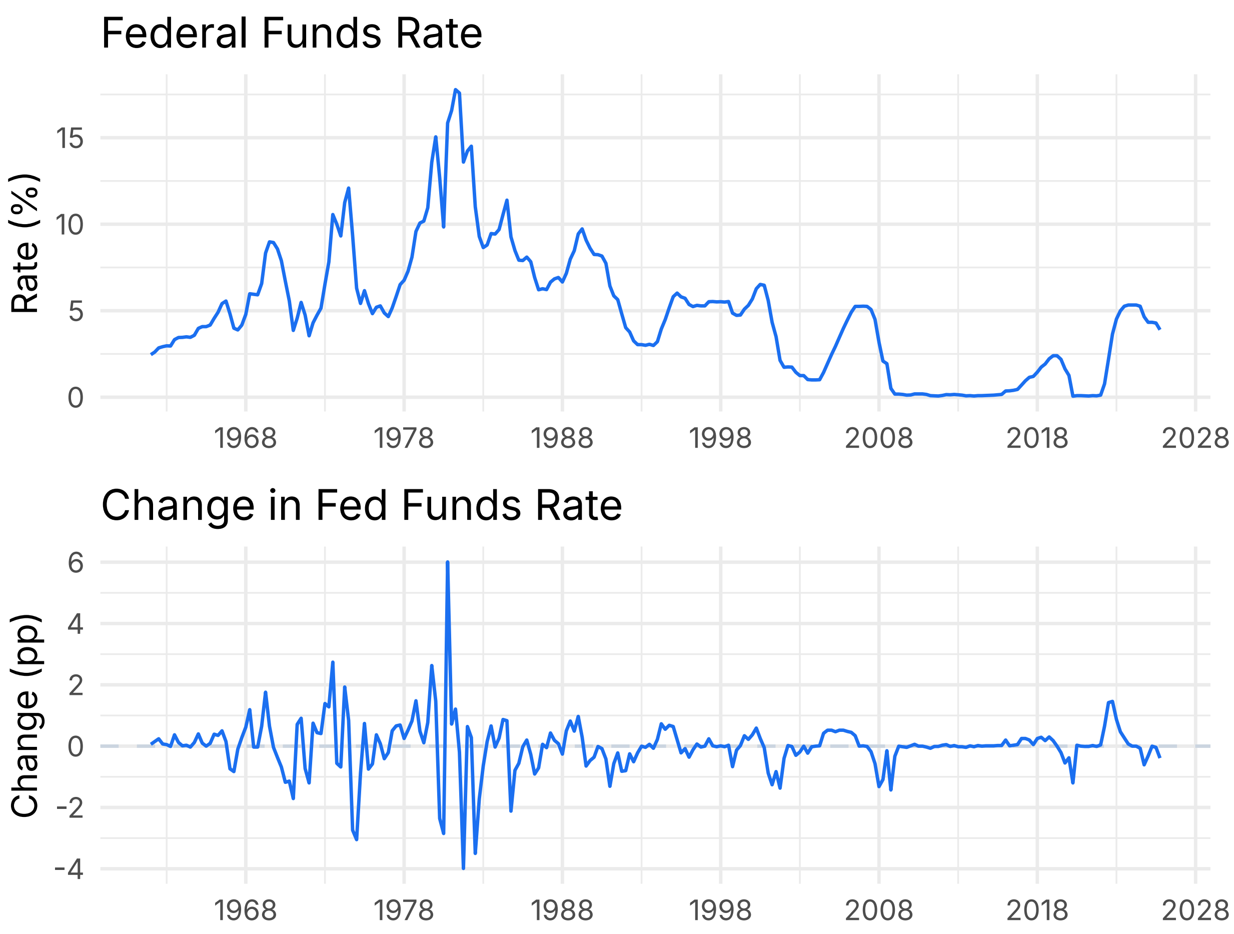
- **CPI level** (top): Consumer Price Index, 1982–84 = 100
- **Inflation** (bottom): annualised quarterly inflation rate ($400 \times \Delta \ln CPI_t$)
- Quarterly data, 1962–2025

Unemployment Rate



- **Unemployment rate** (top): civilian unemployment rate (%)
- **Change in unemployment** (bottom): first difference (Δune_t)
- Quarterly data, 1962–2025

Federal Funds Rate

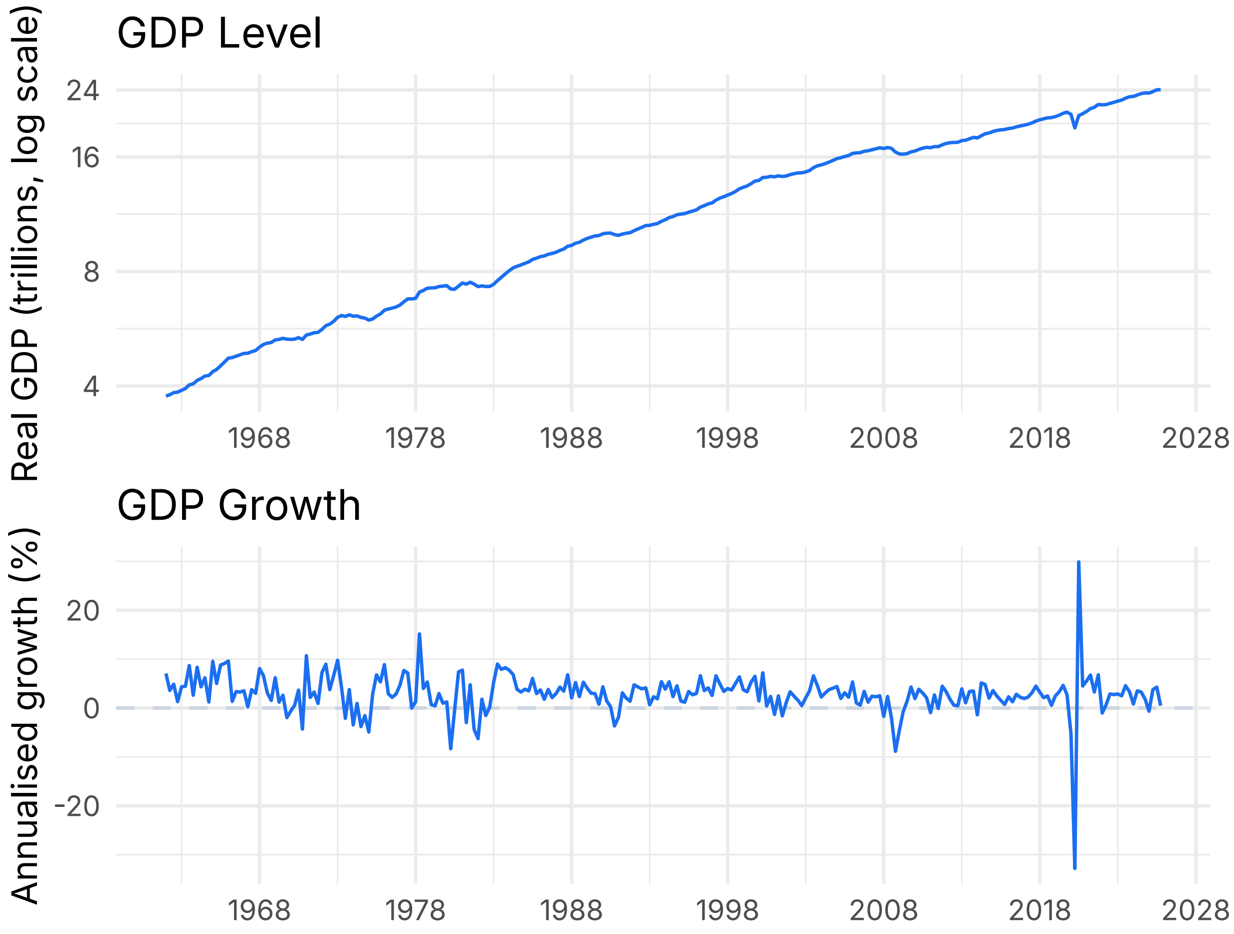


Source: FRED, series FEDFUNDS.

- **Federal Funds Rate** (top): the overnight interbank lending rate set by the Fed
- **Change in FFR** (bottom): first difference (ΔFFR_t)
- Quarterly data, 1962–2025

Exercises

Exercise 1 | GDP Level vs. GDP Growth



Source: FRED, series GDPC1.

a) The GDP level and GDP growth rate are plotted on the left. Which looks stationary? What visual features of each series inform your assessment?

Exercise 1 | GDP Level vs. GDP Growth

b) Augmented Dickey-Fuller tests are run on both series:

```
ADF test on GDP level:
  Dickey-Fuller = -0.63,   Lag order = 6,   p-value = 0.98
  => Fail to reject H0: unit root present.

ADF test on GDP growth:
  Dickey-Fuller = -6.28,   Lag order = 6,   p-value = 0.01
  => Reject H0: unit root absent.
```

Are these results consistent with the visual assessment in a)?

Exercise 1 | GDP Level vs. GDP Growth

c) The ADF in b) used the **intercept + trend** specification. Compare results from both specifications on GDP level:

```
ADF (intercept only):
    tau =  2.67,  5% c.v. = -2.87  =>  Fail to reject H0.
ADF (intercept+trend):
    tau = -0.87,  5% c.v. = -3.42  =>  Fail to reject H0.
=> Both fail to reject H0: unit root present under both specifications.
```

The intercept-only tau is positive, while the other is negative. What does this mean? Which specification is appropriate, and why?

Exercise 2 | Spurious Regression

a) A researcher regresses the GDP **level** on the CPI **level**:

```
              Estimate  Std. Error  t value  Pr(>|t|)
(Intercept)    1995.6        86.8     23.0   < 0.001 ***
cpi              68.5         0.5    133.7   < 0.001 ***

R² = 0.986
```

The R^2 is 0.99 and the t-statistic is 133.7. Is this evidence of a genuine economic relationship between the price level and real GDP?

Exercise 2 | Spurious Regression

b) The same researcher redoes the analysis using GDP **growth** and **inflation**:

```
              Estimate  Std. Error  t value  Pr(>|t|)
(Intercept)      2.712       0.420     6.45   < 0.001 ***
inf              0.058       0.088     0.66     0.509

R² = 0.002
```

How do R^2 and the p-value change relative to a)? What is the correct conclusion about the growth–inflation relationship?

Exercise 2 | Spurious Regression

c) What does the spurious regression example tell us about how to read empirical papers that use macroeconomic data in levels?

Exercise 3 | Testing Inflation: How Many Differences?

a) ADF tests on inflation and on the **change** in inflation (Δinf_t):

```
ADF on inflation (intercept only):
    tau = -4.77,  5% c.v. = -2.87  =>  Reject H0: stationary.

ADF on Δinflation (intercept only):
    tau = -6.45,  5% c.v. = -2.87  =>  Reject H0: stationary.
```

The ADF rejects the unit root for inflation. Yet the CPI plot shows inflation trending strongly through the 1970s. Should we trust the test and treat inflation as $I(0)$?

Exercise 3 | Testing Inflation: How Many Differences?

b) Suppose you want to forecast inflation using lagged unemployment as a predictor. Given the uncertainty about inflation's integration order established in a), what is the safest modelling strategy and why?

Exercise 4 | Classifying Series by Integration Order

ADF tests (both specifications) are run on five US macroeconomic series (quarterly, 1962–2025), which are shown in the Data Overview above. Results are in the table below:

Series	tau (intercept only)	tau (intercept+trend)	Verdict
Real GDP (level)	2.67	−0.87	$I(1)$
GDP Growth	−10.09	−10.35	$I(0)$
CPI (level)	3.21	−0.72	$I(1)$
Inflation	−4.77	−5.16	$I(0)$
Fed Funds Rate	−2.55	−3.28	$I(1)$

Exercise 4 | Classifying Series by Integration Order

a) For each series, explain **why** the tau statistics and specification choice lead to that verdict.

Exercise 4 | Classifying Series by Integration Order

b) For each $I(1)$ series, what is the standard transformation to achieve stationarity? Why is that particular transformation used rather than simply subtracting the sample mean?

Exercise 4 | Classifying Series by Integration Order

c) Both Real GDP level and CPI level have **positive** tau statistics under the drift specification (tau = 2.67 and tau = 3.21 respectively). What does a positive tau mean, and why does this arise specifically for these two series?

Discussion

1. The ADF test has H_0: unit root. Why does failing to reject H_0 not prove that the series has a unit root? When should you be especially skeptical of a non-rejection?

Discussion

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2. Looking ahead to the research project: how would you check whether the Portuguese electricity consumption series is stationary?