Reassessing the Role of Capital in the Dynamics of the Labor Share

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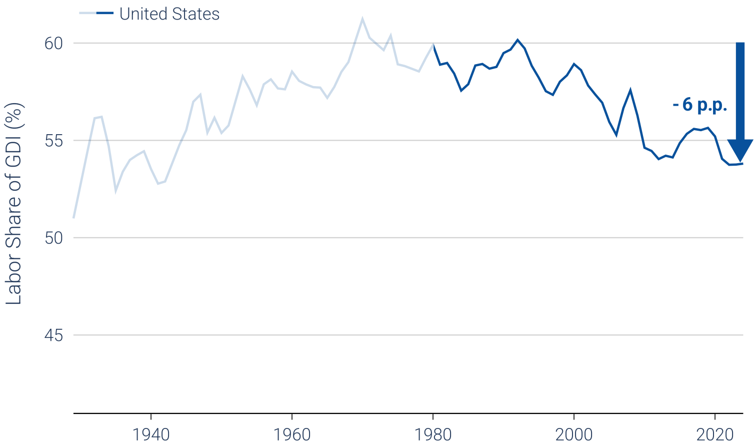
Introduction

The Decline of the Labor Share in the US



Source: Bureau of Economic Analysis (2024)

Introduction

Motivation

- Consensual: 6 p.p. decline since the 1980s across developed countries (Karabarbounis, 2024)
- The labor share is central to research on inequality
- Concerns about:
  - Living standards of the poor
  - Social stability
  - Economic growth sustainability

Introduction

Literature Focus on the Primary Mechanism

Several declining mechanisms (Grossman and Oberfield, 2022)

- Technological progress (Acemoglu and Restrepo, 2020; Karabarbounis and Neiman, 2014)
- Globalization (Elsby *et al.*, 2013)
- Market Power (Autor *et al.*, 2020; Barkai, 2020; De Loecker *et al.*, 2020)
- Labor force composition (Acemoglu and Restrepo, 2022; Glover and Short, 2020; Grossman, Helpman, *et al.*, 2021)

Introduction

Literature Focus on the Primary Mechanism

But...

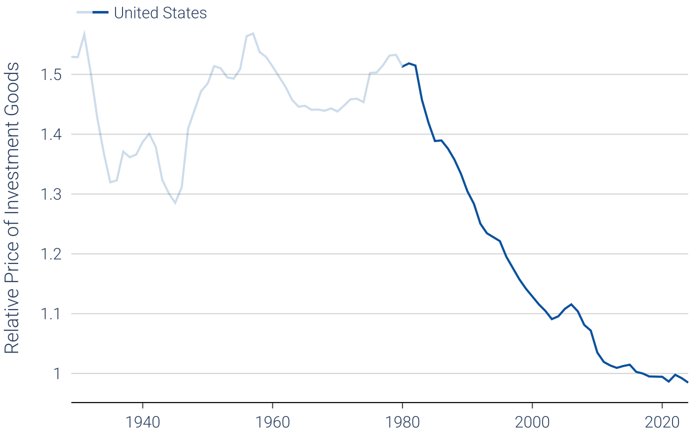
- Overestimated partial effects for a rather stable historical trend (Harrison, 2024)
- Intellectual Property Products capitalization explains it all (Koh *et al.*, 2020)

Introduction
This paper

- Labor share stability demands offsetting forces
- Investment-embodied technological progress decreases the relative price of investment goods
- The resulting inputs reallocation can act as a countervailing factor of the decline of the labor share
- When there are sectoral differences in capital-output elasticities

Introduction

The Decline of the Relative Price of Investment Goods



Source: Bureau of Economic Analysis (2024)

Introduction

The Decline of the Relative Price of Investment Goods

- Drivers: equipment and intellectual property Details
- Evidence of investment-embodied technological progress (Greenwood *et al.*, 1997; Hubmer, 2023; Karabarbounis and Neiman, 2014; Solow *et al.*, 1960)
- It plays a role on the decline of the labor share when $\sigma_{K,L} > 1$ (Karabarbounis and Neiman, 2014; Lawrence, 2015)

Introduction

Approach

- Standard two-sector growth model
- Three key assumptions
  1. Unitarian capital-labor elasticity of substitution
  2. Different capital-output elasticities across sectors
  3. Investment-embodied technological progress

Introduction
Main Findings

1. Labor share changes are transitional
2. Triggered by capital redistribution across sectors
3. Require different sectoral capital-output elasticities
4. Occur despite a unitary capital-labor elasticity of substitution
5. Increases can happen even when $r > g$ (Piketty, 2014; Piketty and Zucman, 2014)

The Model

The Model

Setup: Households Preferences

Preferences are described by a CES utility function:

$$U = \sum_{t=0}^{+\infty} \beta^t \frac{(C_t)^{1-\phi}}{1-\phi}, \qquad \phi^{-1} \geq 0, \quad 0 < \beta < 1 \qquad (1)$$

The Model

Setup: Labor and Capital

- Labor supply is exogenous and homogeneous:

$$L_t = L_0 \left(1 + g^L\right)^t, \qquad g^L \geq 0 \tag{2}$$

- Capital is homogeneous and evolves according to:

$$K_{t+1} = K_t(1 - \delta) + I_t, \qquad 0 < \delta < 1 \tag{3}$$

The Model

Setup: Production

- Two sectors: consumption (C) and investment (I)
- Both use two inputs: capital (K) and labor (L)
- In each sector $j \in \{C, I\}$:

$$Y_t^j = (K_t^j)^{\alpha^j} (A_t^j L_t^j)^{1-\alpha^j} \qquad 0 < \alpha^j < 1 \tag{4}$$

with $\alpha^j \neq \alpha^{-j}$

The Model

Setup: Technological Progress

- Harrod neutral technological progress:

$$A_t^j = (1 + g^{A^j})^t A_0^j, \qquad g^{A^j} \geq 0 \tag{5}$$

- Investment-embodied technological progress means: $g^{A^I} > g^{A^C}$

The Model

Setup: Resources Constraints for Inputs and Outputs

- Resources constraints for inputs:

$$K_t = K_t^C + K_t^I \tag{6}$$

$$L_t = L_t^C + L_t^I \tag{7}$$

- Resources constraints for outputs:

$$Y_t^C = C_t \tag{8}$$

$$Y_t^I = I_t \tag{9}$$

The Model

Setup: Some Convenient Definitions

- Share of capital allocated to sector I:

$$s_t^K \equiv K_t^I / K_t$$

- Share of labor allocated to sector I:

$$s_t^L \equiv L_t^I / L_t$$

- Capital per effective worker:

$$k_t \equiv K_t / \left(A_t^I L_t\right)$$

The Model

Planner Problem

- A benevolent social planner chooses the path for:
  - Shares of inputs $\{s_t^K, s_t^L\}_{t=0}^{+\infty}$
  - Capital per effective worker $\{k_{t+1}\}_{t=0}^{+\infty}$
- That maximize the utility function (1)
- Subject to the resources constraints (6)–(9)
- Given: β, ϕ, α^C, α^I, δ, g_{A^C}, g_{A^I}, and g^L, and K_0

The Model

Solution: Planner Problem

- The problem admits a single solution to s_t^K, s_t^L and k_{t+1} Details
- Property: $\alpha^C = \alpha^I = \alpha \Rightarrow s_t^L = s_t^K$
- With these values we can determine all the quantities
  - Inputs in each sector: K_t^C, K_t^I, L_t^C and L_t^I
  - Outputs Y_t^C and Y_t^I
  - Allocations C_t and I_t

The Model

Solution: Decentralized Competitive Equilibrium

- Identical conditions for quantities and conditions for prices, namely:

$$\frac{1}{q_t} \equiv \frac{P_t^I}{P_t^C} = \frac{\alpha^C}{\alpha^I}\left(\frac{A_t^C}{A_t^I}\right)^{1-\alpha^C}(k_t)^{\alpha^C-\alpha^I} \times \\ \times \left[\left(\frac{s_t^K}{s_t^L}\right)^{1-\alpha^I} \Big/ \left(\frac{1-s_t^K}{1-s_t^L}\right)^{1-\alpha^C}\right] \tag{10}$$

- Notice that: $\alpha^C = \alpha^I = \alpha \Rightarrow 1/q_t = (A_t^C/A_t^I)^{1-\alpha}$

Details

The Model

Determinants and Long-Run Behavior of The Labor Share

$$m_t^L \equiv \frac{w_t^I L_t}{Y_t} = \frac{\alpha^I(1-\alpha^C) + (\alpha^C - \alpha^I)s_t^K}{\alpha^I + (\alpha^C - \alpha^I)\, s_t^K} \tag{11}$$

- Notice that: $\alpha^C = \alpha^I = \alpha \Rightarrow m_t^L = 1 - \alpha$
- A Balanced Growth Path in this economy requires s_t^K to be constant over time Details
- So, the labor share is constant in the long-run

The Model

Transition Dynamics of The Labor Share to the BGP Level

- Along a transition path of s^K to the steady state:

$$\frac{\mathrm{d}m_t^L}{\mathrm{d}s_t^K} = \frac{(\alpha^c - \alpha^I)\,\alpha^I\alpha^c}{\left(\alpha^c s_t^K + \alpha^I\,(1 - s_t^K)\right)^2} \tag{12}$$

- Hence:
  - if $\alpha^c > \alpha^I$: m_t^L moves in the same direction of s_t^K
  - if $\alpha^c < \alpha^I$: m_t^L moves in the opposite direction of s_t^K

Calibration

Baseline Parameters

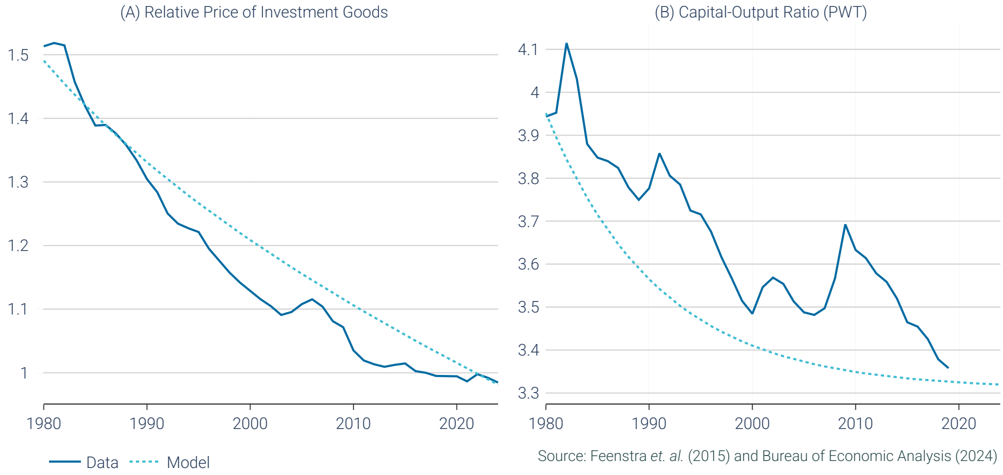
Parameter	Value	Source/Calibration target
β	0.96	Prescott (1986)
ϕ^{-1}	0.65	Vissing-Jørgensen (2002)
δ	3.8%	1980–2024 average from Feenstra *et al.* (2015)
g^L	0.9%	1980–2024 g^{Pop} from US Census Bureau (2025)
α^C	50%	Basu *et al.* (2013) and 1980–2024 q_t^{-1} evolution
α^I	26%	from US Bureau of Economic Analysis (2024)
g^{A^C}	1.0%	1980–2024 q_t^{-1} evolution from US Bureau of
g^{A^I}	2.7%	Economic Analysis (2024)

BGP values

Results

Results

Calibration Targets



Source: Feenstra *et. al.* (2015) and Bureau of Economic Analysis (2024)

Results

The Transition Path of k_{t+1}

- Initial excess of capital enables high consumption and low investment
  - $s_0^K = 11.74\% < 14.47\% = s_*^K$
  - $s_0^L = 27.46\% < 32.5\% = s_*^L$
- Then, depreciation and technological progress trigger the transfer of inputs from sector C to sector I Shares Plot

Results

The Transition Path of k_{t+1}

- Sector switching always equalizes the Marginal Rate of Technical Substitution across sectors

$$\frac{1-\alpha^C}{\alpha^C}\frac{\alpha^I}{1-\alpha^I} = \frac{1-s_t^K}{1-s_t^L}\Bigg/\frac{s_t^K}{s_t^L} \tag{13}$$

- Both sectors become more capital intensive

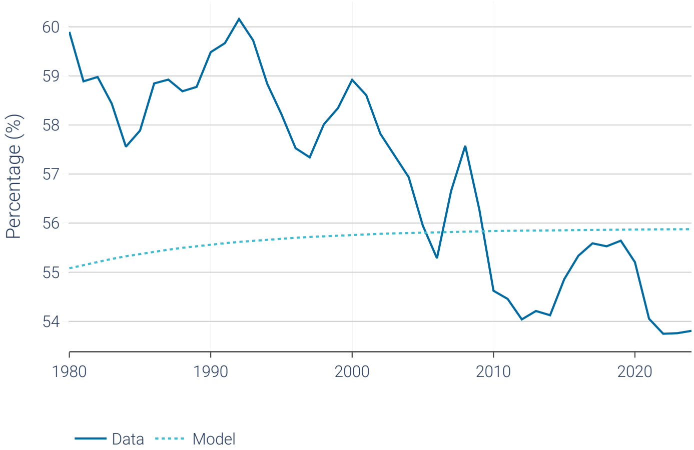
Results

The Transition Path of the Labor Share

- Recall that: $m_t^L \equiv (w_t^I L_t) / Y_t$
- Real wage w^I increases due to higher capital intensity
- Impact on total output $Y_t \equiv q_t Y_t^C + Y_t^I$ is unclear
  - Y_t^C decreases and Y_t^I increases due to input sector switching
  - q_t^{-1} decreases (mainly) because $g_{A^I} > g_{A^C}$ Eq. Condition

Results

The Labor Share



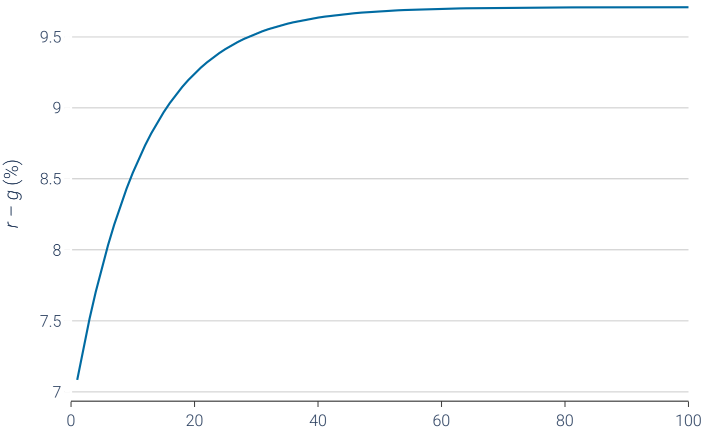
Results

The Labor Share

- We already knew that m_t^L and s_t^K move in opposite directions when $\alpha^c > \alpha^I$
- The increase in the real wage dominates over the increase in aggregate output
- The labor share increases around +1p.p.
- According to Piketty, this requires $r - g < 0$

Results

Piketty's $r > g$ Channel is Missing



Concluding Remarks

Concluding Remarks

- Long-run labor share unaffected
- Short-term labor share changes driven by sectoral capital-output elasticities
- An increase in the labor share is expected when:
  - Consumption goods sector has a higher capital-output elasticity than investment goods
  - Capital per effective worker is above the steady state

Concluding Remarks

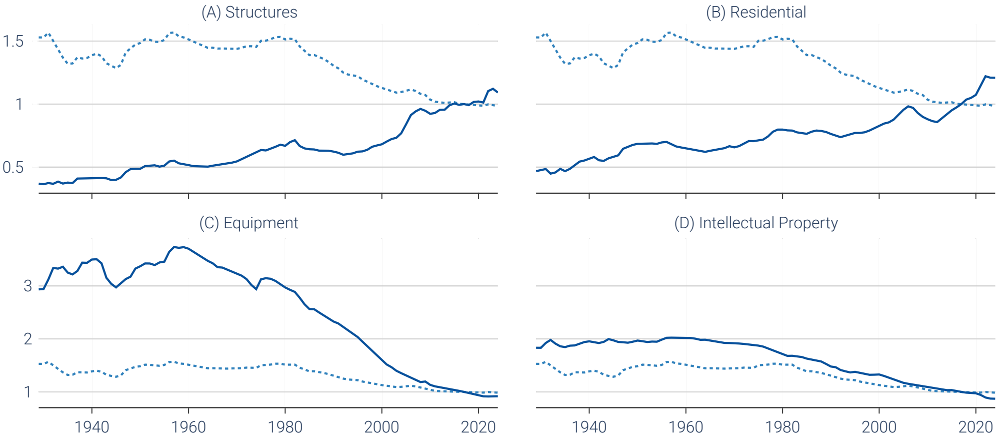
- Mechanism does not require $\sigma_{L,K} > 1$
- The increase happens despite $r > g$
- Acts as a countervailing mechanism to the observed decline in labor share over the past four decades

Open Questions

- What does $\alpha^c > \alpha^I$ mean? And what is the exact magnitude of the difference?
- Why was the capital per effective worker above the steady state in the 80s?
- What are the effects of other declining mechanisms?
- How does the welfare distribution change with the relative price?

Appendixes

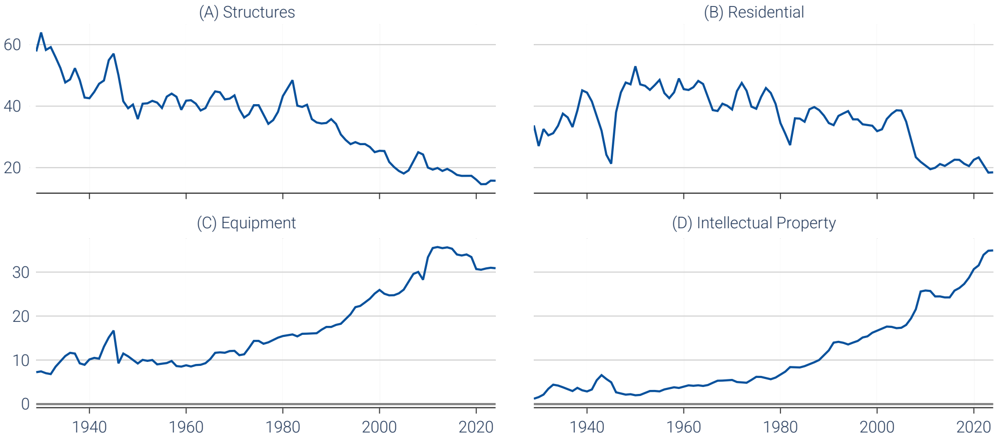
Relative Prices of Investment by Type of Asset



Source: Bureau of Economic Analysis (2024)

Shares of Investment by Type of Asset

Back to Introduction



Source: Bureau of Economic Analysis (2024)

First Best Solution

Equilibrium Condition for Variable s_t^K

$$
\begin{aligned}
&\left(1+g^{A^C}\right)^{(1-\alpha^C)(\phi-1)}\left(1+g^{A^I}\right)^{1-\alpha^C(1-\phi)}\left(1+g^L\right)^{\phi} = \\
&= \left(\frac{k_t}{k_{t+1}}\right)^{\alpha^I-\alpha^C(1-\phi)}\left(\frac{s_t^L}{s_{t+1}^L}\right)^{1-\alpha^I}\left(\frac{s_{t+1}^K}{s_t^K}\right)^{1-\alpha^I}\times \\
&\times\left(\frac{1-s_{t+1}^L}{1-s_t^L}\right)^{(1-\alpha^C)(1-\phi)}\left(\frac{1-s_t^K}{1-s_{t+1}^K}\right)^{1-\alpha^C(1-\phi)}\times \\
&\times\beta\left[\alpha^I\left(k_{t+1}\right)^{\alpha^I-1}\left(\frac{s_{t+1}^L}{s_{t+1}^K}\right)^{1-\alpha^I}+1-\delta\right] \qquad (14)
\end{aligned}
$$

First Best Solution

Equilibrium Condition for Variable s_t^L and k_{t+1}

$$s_t^L = \left[\frac{\alpha^I}{1-\alpha^I} \frac{1-\alpha^C}{\alpha^C} \frac{1-s_t^K}{s_t^K} + 1 \right]^{-1} \tag{15}$$

$$k_{t+1} = k_t \frac{1-\delta}{(1+g_{A^I})(1+g_L)} + \frac{\left(s_t^K k_t\right)^{\alpha^I} \left(s_t^L\right)^{1-\alpha^I}}{(1+g_{A^I})(1+g_L)} \tag{16}$$

First Best Solution
Transversality Condition

$$\lim_{t \to +\infty} \frac{\beta^t}{C_t^{\phi}} \frac{\alpha^C}{\alpha^I} (k_t)^{\alpha^C - \alpha^I} \left(\frac{s_t^K}{s_t^L} \right)^{1-\alpha^I} \left(\frac{1 - s_t^L}{1 - s_t^K} \frac{A_t^C}{A_t^I} \right)^{1-\alpha^C} K_{t+1} = 0 \tag{17}$$

First Best Solution

Back to the Model

Equilibrium Conditions for the Original Variables

$$C_t = \left((1 - s_t^K)K_t\right)^{\alpha^C} \left((1 - s_t^L)A_t^C L_t\right)^{1-\alpha^C} \quad (18)$$

$$I_t = \left(s_t^K K_t\right)^{\alpha^I} \left(A_t^I s_t^L L_t\right)^{1-\alpha^I} \quad (19)$$

$$L_t^I = s_t^L L_t \quad (20)$$

$$K_t^I = s_t^K K_t \quad (21)$$

$$L_t^C = L_t - L_t^I \quad (22)$$

$$K_t^C = K_t - K_t^I \quad (23)$$

$$K_{t+1} = k_{t+1} A_{t+1}^I L_{t+1} \quad (24)$$

Decentralized Economy

Firms Problem

- Firms in each sector $j \in \{C, I\}$ maximize profits:

$$\Pi_t^j = P_t^j Y_t^j - W_t L_t^j - R_t K_t^j \tag{25}$$

- Subject to production technologies in (4)
- Given:
  - Price of its own output P_t^j
  - Nominal cost rate of inputs: W_t and R_t

Decentralized Economy

Households Problem

- Households maximize utility in (1)
- Subject to a budget constraint:

$$q_t C_t + I_t \leq w_t^I L_t + r_t^I K_t \tag{26}$$

- Given:
  - Relative price $q_t \equiv P_t^C / P_t^I$
  - Real returns to inputs: $w_t^I \equiv W_t / P_t^I$ and $r_t^I \equiv R_t / P_t^I$

Decentralized Competitive Equilibrium

Definition

- Sequence for:
  - Inputs $\{L_t^C, L_t^I\}_{t=0}^{+\infty}$ and $\{K_t^C, K_t^I, K_{t+1}\}_{t=0}^{+\infty}$
  - Outputs $\{C_t, I_t\}_{t=0}^{+\infty}$
  - Real returns to inputs $\{w_t^I\}_{t=0}^{+\infty}$ and $\{r_t^I\}_{t=0}^{+\infty}$
  - Relative price $\{q_t\}_{t=0}^{+\infty}$
- So that:
  - Firms solve their optimization problem
  - Households solve their optimization problem
  - Input and output markets clear according to (6)–(9)

Decentralized Competitive Equilibrium

Back to the Model

Solution

- Same equilibrium conditions as in the First Best Solution for inputs and outputs
- Equilibrium conditions for real returns no inputs:

$$r_t^I = \alpha^I \left(\frac{s_t^L}{s_t^K} \right)^{1-\alpha^I} (k_t)^{\alpha^I - 1} \tag{27}$$

$$w_t^I = (1-\alpha^I) A_t^I \left(\frac{s_t^K}{s_t^L} \right)^{\alpha^I} (k_t)^{\alpha^I} \tag{28}$$

Balanced Growth Path

Condition for s_*^L

- Assume that $k_t = k_*$ and $s_t^K = s_*^K$, for some t
- Then, a solution for the Planner Problem exists if and only if:

$$s_*^L = \left[\frac{\alpha^I}{1-\alpha^I} \frac{1-\alpha^C}{\alpha^C} \frac{1-s_*^K}{s_*^K} + 1 \right]^{-1} \tag{29}$$

Balanced Growth Path

Condition for s_*^K

$$s_*^K = \left[\frac{(1+g_{A^C})^{(1-\alpha^C)(\phi-1)} (1+g_{A^I})^{1-\alpha^C(1-\phi)} (1+g_L)^{\phi}}{\beta \alpha^I \left((1+g_{A^I})(1+g_L) - (1-\delta)\right)} - \right.$$
$$\left. - \frac{1-\delta}{(1+g_{A^I})(1+g_L) - (1-\delta)} \right]^{-1} \tag{30}$$

$$k_* = \left(\frac{(s_*^K)^{\alpha^I} (s_*^L)^{1-\alpha^I}}{(1+g_{A^I})(1+g_L) - (1-\delta)} \right)^{\frac{1}{1-\alpha^I}} \tag{31}$$

Balanced Growth Path

Back to the Labor Share

Characterization

$$g_{K^I} = g_{K^C} = g^K = g^I = (1 + g_{A^I})(1 + g^L) - 1 \tag{32}$$

$$g_{L^I} = g_{L^C} = g^L \tag{33}$$

$$g^C = \left(\frac{1 + g_{A^I}}{1 + g_{A^C}}\right)^{\alpha^C} (1 + g_{A^C})(1 + g^L) - 1 \tag{34}$$

$$g_{r^I} = 0 \tag{35}$$

$$g_{w^I} = g_{A^I} \tag{36}$$

$$g^q = \left(\frac{1 + g_{A^I}}{1 + g_{A^C}}\right)^{1-\alpha^C} - 1 \tag{37}$$

Balanced Growth Path

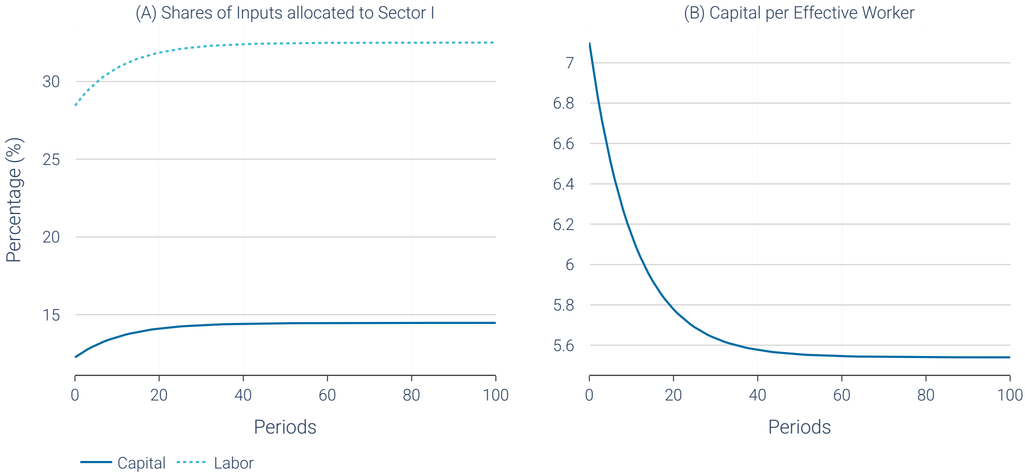
Back to Calibration

Values Resulting from Calibration

Variable	Value
s_*^K	14.47%
s_*^L	32.50%
k_*	5.54
g_*^C	2.76%
$g_*^I = g_*^Y = g_*^K = g_*^{K^C} = g_*^{K^I}$	3.62%
$g_*^{w^I}$	2.70%
$g_*^{r^I}$	0.00%

Simulation of the Core Variables

Back to Results



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